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# Nonequilibrium phase transitions in a model with social influence of inflexible units

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**Abstract**

In many social, economical and biological systems, the evolution of the states of interacting units cannot be simply treated with a physical law in the realm of traditional statistical mechanics. We propose a simple binary-state model to discuss the effect of the inflexible units on the dynamical behavior of a social system, in which a unit may have a chance to keep its state with a probability $1-q$ even though its state is different from those of the majority of its interacting neighbors. It is found that the effect of these inflexible units can lead to a nontrivial phase diagram.

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## 1. Introduction

Recently, the dynamical behavior of social systems has attracted much attention in complex science. The Axelrod's model of social interaction is proposed to understand the formation of cultural domains [1–4] and a voter model is employed to study the formation of public opinion [5–7]. In these models, the state of an individual is determined by one of the neighbors randomly. Moreover, several models are defined on so-called simple *majority rule* [8–10], which roots in the physics law in the field of classical statistical mechanics. In fact, the minority sometimes takes an important role in the evolutions of many real social systems, such as the transmission of cultural traits, the formation of public opinion and the choice of an agent's policy. In order to investigate the effect of the minority, a majority–minority model is proposed where the states of all of sites of a group are determined by the local majority with a probability $k$ or by the local minority with a probability $1-k$ and a phase transition appears when $k$ changes [11, 12].

In a real social life, the interactions between neighbors will not follow the traditionally physical laws simply. The existence of a stubborn unit is a kind of special social phenomena, such as not every kind of culture can change its features undoubtedly when it is surrounded by other cultures. Such a refusal dynamics effect arising from local interactions is also found

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in public opinion formation [13]. It is very interesting to explore the effect of the stubborn units on the evolutionary behavior in a social system.

In order to understand such kind of social phenomena, we introduce an inflexible psychology of an individual in a social system. In our dynamical rule, when the state of a site is the same as most of its mates, a change of its state will satisfy a majority rule [8]. When the state of a unit is different from the most of its mates, its state may change to follow the surrounding effect with a probability $q$ $(0 < q \leqslant 1)$ or can have a tendency to keep its state with a probability $1 - q$. This inflexible mechanism is different from the simple majority rule and minority rule. With the introduction of this competition influence, it is shown that the effect of the inflexible units can lead to a nontrivial phase diagram in which abundant phase transition behaviors are exhibited.

## 2. Model

In a two-dimensional lattice surface, the state of each site may represent a viewpoint or a kind of culture, etc. The state of every one evolves according to a social process, whose dynamic behavior is determined not only by other correlative surrounding effects but also by its own character.

To illustrate the cases simply, first, we suppose that an individual in the system can only stay in one of two possible states, $\sigma_i \in \{+1, -1\}$. We define a function $W(\sigma_i)$ to describe the interaction with its neighbors. $W(\sigma_i) = 2\sigma_i \sum_{j=1}^{4} \sigma_j$, here $\sigma_j (j = 1, 2, 3, 4)$ are the states of four sites connected to node $i$ and these four sites are called mates of $i$.

We introduce the updating rule of a state. When $W(\sigma_i) > 0, \sigma_i$ has the same state as the most of its mates, $\sigma_i$ changes its state with a probability $\exp[-W(\sigma_i)/T]$, which relies on a temperature-like parameter $T$ as that in a usual physical system. However, when function $W(\sigma_i) < 0, \sigma_i$ is different from most of its mates, we assume that the overturning probability of $\sigma_i$ is not a unit one but a constant $q(0 < q \leqslant 1)$. And when $W(\sigma_i) = 0$, we consider the overturning probability of $\sigma_i$ is equal to $q$ also. Therefore, an individual has an opportunity to keep its state when its state is different from the most of its mates. From the dynamical rule, we can see that, when $q = 1$, the model restores to the Ising model [8]. However, our model is a non-equilibrium model because the overturning probability of a state does not satisfy the detailed equilibrium in the model.

## 3. Simulation results

In order to describe the dynamical behavior of the model, we define a magnetization-like order parameter $m$ as follows: $m = \frac{1}{L^2} \sum_{i=1}^{L^2} \sigma_i$, here $\sigma_i \in \{+1, -1\}$. The lattice size is $L \times L$, and the sum is carried out over all of the sites. Obviously, the system enters into an ordered state when $\langle |m| \rangle$ approaches 1, i.e. all of the sites stay almost in the same state. However, when $\langle |m| \rangle$ reaches zero, it stays in a complete disordered state.

An extensive Monte Carlo numerical simulation has been performed on our model with a random initial condition and a periodic boundary condition in a regular lattice. The lattice sizes are $16 \times 16, 32 \times 32, 64 \times 64$, respectively. The order parameter has been calculated after the system reaches a non-equilibrium stationary state. The calculated results are averaged over $n$ independent runs, where $n \times L^2 \approx 1 \times 10^6$.

As the simulation results shown in figure 1, it is found that the system enters into an ordered state when $T$ is below a critical temperature $T_c(q)$. In the top of figure 1, the change of $\langle |m| \rangle$ denotes a continuous phase transition with $T$ varying for $q = 0.9$, while the simulation results in the bottom of figure 1 disclose that there is a discontinuous phase transition with

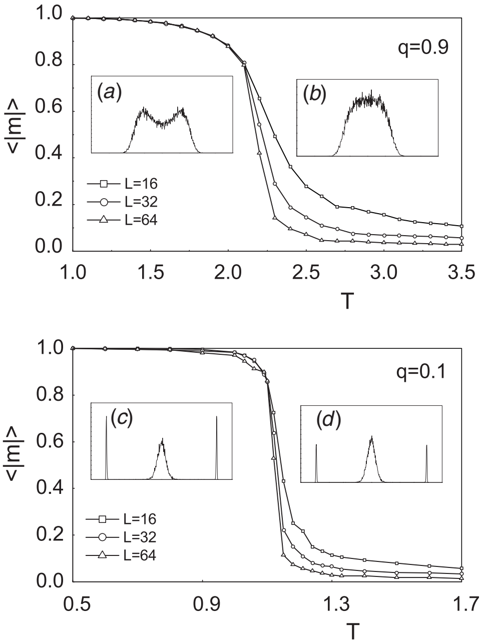


**Figure 1.** The order parameter $|\langle m\rangle|$ against $T$. From figures ($a$) to ($b$), the system exhibits a continuous phase transition from an ordered state to a disordered state for $q = 0.9$. From figures ($c$) to ($d$), the system exhibits a discontinuous phase transition for $q = 0.1$.

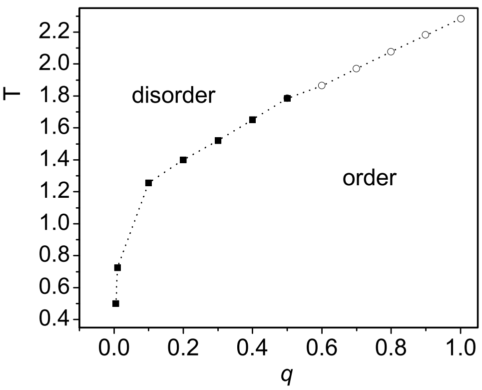


**Figure 2.** Phase diagram of the model in the $q-T$ plane. The system displays a line of a continuous phase transition when $q > q_c$. The open symbols represent continuous phase transitions and the solid ones represent discontinuous phase transitions.

$T$ varying for $q = 0.1$. From the probability density functions (PDF) of the order parameter inserted in figure 1 near the phase transition point, one can distinguish the continuous phase transition and the discontinuous phase transition more clearly. It seems that the inflexible units cause the change of the phase transition.

From the simulation results in figure 2, it is shown that there is a line of continuous phase transition when $q > q_c$ and the phase transition becomes a discontinuous one for $q < q_c$. The point $q_c$ is estimated roughly to be 0.55(5); it is shown that there is a nonequilibrium tricritical point at $q_c$. When $q \to 0$, then phase transition $T_\mathrm{c}$ decreases rapidly.

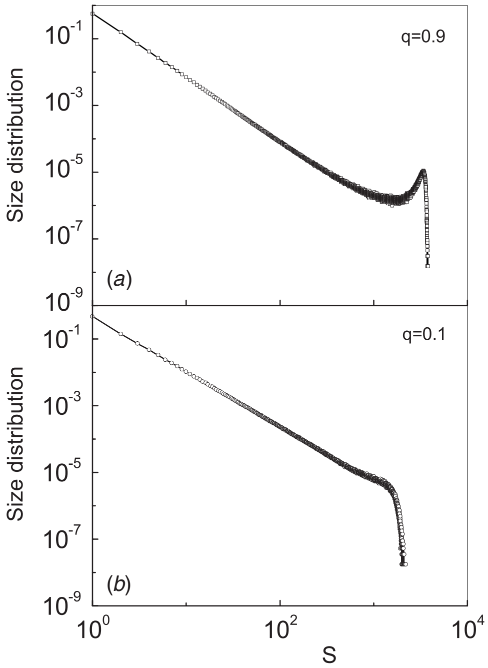


**Figure 3.** Domain size distribution at the critical point. (*a*) The log–log plot of $\rho$ against domain size $s$, $T = 2.18$, $q = 0.9$, the exponent $\tau = 1.94 \pm 0.3$. (*b*) The log–log plot of $\rho$ against domain size $s$, $T = 1.255$, $q = 0.1$, $\tau = 1.647 \pm 0.003$, $L = 64$.

In order to further understand the phase transition behaviors at different $q$ values, we discuss the domain size distribution at the critical point $T_c$. The neighborhood sites in a same state are in a same domain; the number of sites in a domain is defined as the domain size. For $T < T_c$, there are few small size domains and few large size domains; the largest size approaches $L^2$. On the other hand, for $T > T_c$, there are many small size domains. Close to the critical point, it is found that the domain size $s$ distributes in a power law, $\rho(s) \sim s^{-\tau}$ for $s \ll L^2$, where $\rho(s)$ is the probability of a domain size. As shown in figures 3(*a*) and (*b*), we can calculate the exponent $\tau$, $\tau = 1.94 \pm 0.03$ for $q = 0.9$ and $\tau = 1.647 \pm 0.003$ for $q = 0.1$, respectively. Because the domain size distribution is related to the internal spatial structure of a system at the neighbor of the critical point, the difference of exponent $\tau$ indicates a change of the internal structure when $q = 0.9$ and $q = 0.1$.

We can find that, in figure 3(*a*), there is a local maximum probability of the domain for $q = 0.9$ compared to the domain distribution in figure 3(*b*). Small $q$ corresponds to that an individual has a stronger tendency to keep its state and then the interaction between the neighbors decreases and the formation of a large domain becomes difficult. But it is easy for a unit to follow the majority of neighbors to form a larger domain for larger $q$. Therefore, as shown in figure 3(*a*), many more larger domains form gradually for $q = 0.9$ when $T$ decreases to the critical point. Therefore, we can speculate that the existence of the discontinuous phase transition is related to a decrease of the interaction between neighbors along the interface of the different domains.

We employ a finite size scaling analysis to study the critical behavior of the continuous phase transition for $q > q_c$. In the neighborhood of the critical point $T_c$, $\langle |m| \rangle \propto (T_c - T)^{\beta}$

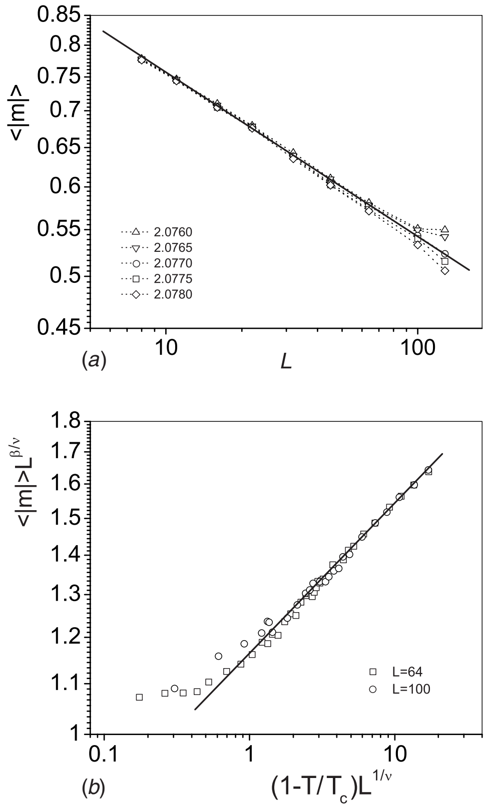


**Figure 4.** (*a*) Double logarithmic plot of the order parameter $\langle|m|\rangle$ versus $L$ for $q = 0.8$. (*b*) A log–log plot of $\langle|m|\rangle L^{\beta/\nu}$ against $(1 - T/T_c)L^{1/\nu}$ for $L = 64$ and $L = 100$. $q = 0.8$.

$(T < T_c)$, where $\beta$ is the order parameter exponent. In addition, when $T$ is near to the critical point $T_c$ of the second-order phase transition, a character length scale $\xi$ denotes the correlation length in space, $\xi \propto (T_c - T)^{-\nu}$ $(T < T_c)$, where $\nu$ is a correlation length exponent in the space direction. At a critical point, various ensemble-averaged quantities depend on the ratio of the system size and the correlation length $L/\xi$. The order parameter $\langle|m|\rangle$ satisfies the scaling form in the neighborhood of the critical point: $\langle|m|\rangle \propto L^{-\beta/\nu} f[(T_c - T)L^{1/\nu}]$ so that at $T_c$, $\langle|m|\rangle \propto L^{-\beta/\nu}$.

In figure 4(*a*), we take a log–log plot of $\langle|m|\rangle$ against $L$ for $q = 0.8$, we can obtain $\beta/\nu = 0.143 \pm 0.001$ for $q = 0.8$. In figure 4(*b*), we have plotted $\langle|m|\rangle L^{\beta/\nu}$ versus $(1 - T/T_c)L^{1/\nu}$ on a double-logarithmic plot for $q = 0.8$. It shows that, with $\beta/\nu = 0.143(1)$ and a choice $\nu = 0.80$, the data for the two lattice sizes are well collapsed on a single curve [14], the slope of the line is $\beta = 0.118 \pm 0.005$. Therefore, we give the exponents $\beta = 0.118(5)$, $\nu = 0.80(1)$ for $q = 0.8$. With the same analysis method, we can obtain $\beta/\nu = 0.134(2)$, $\beta = 0.11(1)$, $\nu = 0.85(2)$ for $q = 0.6$. These critical exponents are different from both mean-field ($\beta = \nu = 1/2$) and exact values ($\beta = 1/8$ and $\nu = 1$) for the equilibrium Ising model in two dimensions. In addition, the critical behavior of these continuous phase transitions also does not belong to any known universality class of nonequilibrium phase transition [15].

## 4. Conclusion

In a real social system, the interactions between individuals are different from those in the traditionally physical system. We propose a simple binary-state model to study the influence of the existence of the stubborn unit in a social system, where a unit has a chance to keep its state with a probability $1-q$ even though its state is different from those of the majority of its interacting neighbors. It is found that the inflexible behavior affects the dynamical behavior seriously. From the numerical simulation results, a nontrivial phase diagram is shown in $q-T$ parameter plane. When $q_c < q < 1$, the system exhibits a continuous phase transition and the transition becomes a discontinuous one when $q$ is below $q_c$. Furthermore, the critical behavior of the nonequilibrium continuous phase transition in the model is different from the well-known universality classes.

## Acknowledgment

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